Professor O. H. Basquin: 'The Bending Moment of a Uniformly Loaded Beam; a New Experimental Demonstration.'

Floyd Field,
Secretary.

DISCUSSION AND CORRESPONDENCE.

CONNECTION BY PRECISE LEVELING BETWEEN THE ATLANTIC AND PACIFIC OCEANS.

To the Editor of Science: In your issue of April 28, 1905, page 673, is an article by Mr. Hayford on 'Connection by Precise Leveling between the Atlantic and Pacific Oceans.' About twenty years ago I wrote to Science in connection with precise leveling over the Alleghanies and the Rocky Mountains, and stated that it might be well to have a systematic determination of bench marks at stated intervals owing to the unrest in the earth's crust. At that time I stated that my work on the corps of the Pennsylvania Railroad had shown me that, however carefully the bench marks might be established at any one time, at the expiration of a comparatively few years there would be a discrepancy between them and the datum plane. The Pennsylvania Railroad has reviewed its bench marks a number of times owing to these discrepancies due to earth motion. The want of agreement, therefore, between the levels of the Atlantic and Pacific Oceans, unless the bench marks were established by surveys which began and ended at exactly the same period throughout the entire distance, might be due to earth movements between the times of the beginning and the end of the survey.

I would again suggest, as I did at my first letter to this paper, that the United States Geological Survey secure not only the lists of bench marks of all railroads, but the variations that have occurred in these bench marks as shown by repeated surveys. If these are carefully tabulated throughout a century, we may obtain important information in regard to the upward and downward crustal movements across the continent.

Edward H. Williams, jr.

SPECIAL ARTICLES.

THE HORIZONTAL PLANE OF THE SKULL AND THE GENERAL PROBLEM OF THE COMPARISON OF VARIABLE FORMS.

In comparative studies of the skull it is customary to select one transversal plane defined by the axis of symmetry with which it is at right angles and by two points, as the standard plane to which the skull is referred. Some authors have made the selection of the two determining points based on morphological considerations, while others have endeavored to determine the physiological horizontal position, determining the latter by two points which are more or less accurately parallel to the direction of horizontal sight.

When this problem is considered from a purely morphological point of view, it will be recognized that there is no justification in selecting arbitrarily two points and disregarding all others, but that the best method of comparison must be based on the assumption that every point of the skull has equal weight and that the nearest approach of all points must be attempted. In this form the problem is applicable to the comparison of all variable forms.

The most favorable superposition of any two forms will be obtained when the sum of the squares of the distances between all pairs of homologous points becomes a minimum. We will refer the body to a system of rectangular coordinates and call x', y' *and* z' the ordinates of a point of the first body, x'', y'' and z'' the ordinates of the homologous point of the second body. By moving the second body by the amounts u, v and w in the direction of the three ordinates, we can modify the relative positions of homologous points without torsion of the body. Then the sum of the squares of the distances of homologous points

$$\Sigma(x'-x''-u)^2+\Sigma(y'-y''-v)^2+\Sigma(z'-z''-w)^2$$

is to be a minimum. Therefore,

$$\Sigma(x'-x''-u)=0.$$

And

$$u=\Sigma(x'-x''),$$

$$v=\Sigma(y'-y''),$$

$$w=\Sigma(z'-z'').$$

Since we may assume the origin of the first system of ordinates arbitrarily, we may take

$$\Sigma x' = \Sigma y' = \Sigma z' = 0;$$

in other words, we take the geometrical center of gravity of the first body as the origin of our system of ordinates. Then

$$\Sigma x'' = \Sigma y'' = \Sigma z'' = 0;$$

i. e., the two bodies must be so placed that their geometrical centers of gravity coincide.

Provided the two forms are symmetrical, this result gives a complete solution of the problem. If the forms are irregular, the degree of torsion must be determined which will give the best result. In most cases the form in question will be symmetrical in at least one direction, so that torsion in one direction only need be considered. Starting with the geometrical center of gravity as the origin of a system of polar coordinates, we have for any given pair of points the coordinates l' and l'' as distances from the center, and a' and a'' as angles with the zero line. If we give the second system of points the torsion ξ, we find that

$$\Sigma\{l'^2 + l''^2 - 2l'l'' \cos(\xi + a'' - a')\}$$

must be a minimum; or

$$\Sigma l'l'' \sin(\xi + a'' - a') = 0,$$

$$tg\xi = -\frac{\Sigma l'l'' \sin(a'' - a')}{\Sigma l'l'' \cos(a'' - a')}.$$

Theoretically, the problem can, therefore, be solved. By using a limited number of well-selected points a good superposition of the two forms can be made.

Experiments, so far as carried out, indicate that alveolar point, nasion, bregma, lambda, basion and pterion give a good superposition of skulls.

It will be noticed that if this method is pursued the arbitrary element in composite drawings or photographs may be eliminated.

FRANZ BOAS.

XUALA AND GUAXULE.

THE location of two Indian villages, Xuala and Guaxule, mentioned in some form by all of the chronicles of Hernando de Soto's wild and unfortunate expedition (1539–41) through the territory now included in the southern states, are important in determining the route of this Spanish adventurer. If the location of these two villages—especially the first—can be determined with reasonable certainty it will enable us to fix the route of the Adelantado with comparative accuracy from his landing place at Tampa Bay, Florida, until he reached the vicinity of Mauvilla in Alabama.

The widest variation in opinion of the numerous authorities touching upon the subject, relates to the position of Xuala; these views, however, may be classed in two unequal groups, as is evident from the following list: The map of Cornelius Wytfliet in his 'Descrip. Ptolemaica (1596)' locates this village on the west side of Savannah River near the head. DeLisle's map (1707?) in French's 'Hist. Coll. La.,' though indefinite, places it west of the Savannah. Later authorities locate it as follows: Pickett ('Hist. Alabama,' I., p. 8); C. C. Jones, Jr. ('Hernando de Soto,' p. 13); Cyrus Thomas (5th 'Ann. Rep. Bur. Eth., p. 95); and Theodora Irving ('Hist. Cong. Florida,' II., p. 8), all locate it west of the upper Savannah in Nacooche valley, Habersham County, Georgia, or in that immediate vicinity. Mr. James Mooney (19th 'Ann. Rep. Bur. Eth.,' pt. 1, p. 195) and Woodbury Lowery ('Spanish Settlements within the United States,' p. 230, in the text, but not on the map) locate it in the 'piedmont' region of North Carolina, about the head of Broad river—which would be about Henderson County. Gilmore Shea in his article entitled 'Ancient Florida,' in Justin Winsor's 'Narrative and Critical History of America, II.,' follows, in this part of De Soto's route, the course given by C. C. Jones, Jr. Buckingham Smith on the map in his 'Narrative of De Soto' (Bradford Club Series, V., pl. 5) places Xuala about Habersham County, Georgia, but locates Guaxule to the northwest, apparently about Towns County of the same state, or possibly over the line, in Tennessee. Although Shipp ('De Soto and Florida') does not locate Xuala, he places Guaxule in Bartow County, Georgia, thus agreeing substantially with Pickett, Jones and